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MATHS - II
HAND-BOOK.

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Chapter-1 – Differential Equation

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* **Methods to solve diff equation:**

1] Variable seperable form.

2] Diff. Equation Reducible to variable seperable form:

a) if in form $\frac{dy}{dx}$, substitute $\frac{y}{x}=u$, $y=ux$.

b) if in form $\frac{dx}{dy}$, substitute $\frac{x}{y}=u$, $x=uy$.

3] Homogenous differential equation:

substitute $y=ux$.

4] Non homogenous diff. equation.

a) if $\frac{a_1}{a_2}=\frac{b_1}{b_2}$ $\therefore lx+my=u$ (substitution)

b) if $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$ Substitute $X=x+h$ $Y=y+k$

$$\therefore \frac{dy}{dx}=\frac{dY}{dX}$$

5] Exact diff. equation:-

Consider equation $M(x,y)dx+N(x,y)dy=0$

$$\therefore Mdx+Ndy=0$$

$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$, then

$$\int Mdx+\int Ndy=c$$

N= not containing terms having x.

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6] Non Exact diff Equation

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

Rules for finding integrating factors.

a] if D.E is homogeneous:

$$I.F = \frac{1}{xM + y.N}$$

b] if D.E is in form $y.f_1(xy)dx + x.f_2(xy)dy = 0$ then

$$I.F = \frac{1}{Mx - Ny}.$$

c] if $\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = f(x)$ then

$$I.F = e^{\int f(x)dx}$$

d) if $\dfrac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = f(y)$ then

$$I.F = e^{\int f(y)dy}$$

7] Linear diff equation:-

Form $= \frac{dy}{dx} + Py = Q$

$$\therefore I.F = e^{\int P dx}$$

Soln Method $= y e^{\int P dx} = \int Q \cdot I.F \, dx + c$

8] Reducible to linear form:-

a) Bernoulli's Diff. Equation: $\frac{dy}{dx} + P.y = Q.y^n$

Soln Method $= \frac{du}{dx} + (1-n)P.u = (1-n)Q$ where $y^{1-n} = u$

b] Equations of form $f'(y)\frac{dy}{dx} + Pf(y) = Q$

Substitute $f(y) = u$

$$\therefore f'\frac{dy}{dx} = \frac{du}{dx}.$$

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USEFUL FORMULAE

1.	$\int e^{at} \sin bt \, dt = \dfrac{e^{at}}{a^2 + b^2} (a \sin bt - b \cos bt)$
2.	$\int e^{at} \cos bt \, dt = \dfrac{e^{at}}{a^2 + b^2} (a \cos bt + b \sin bt)$
3.	$\int e^{at} \sin bt \, dt = \dfrac{e^{at}}{\sqrt{a^2 + b^2}} \sin (bt - \phi)$, where $\phi = \tan^{-1}\left(\dfrac{b}{a}\right)$
4.	$\int e^{at} \cos bt \, dt = \dfrac{e^{at}}{\sqrt{a^2 + b^2}} \cos (bt - \phi)$, where $\phi = \tan^{-1}\left(\dfrac{b}{a}\right)$

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Chapter 2 – Applications of Differential Equation

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1] Newton's law of cooling :-

$$\frac{d\theta}{dt} = -K(\theta-\theta_0)$$

θ_0 = room temperature   K = constant

2] Rectilinear Motion :-

m = mass of body   v = velocity
F = Force   x = displacement
t = time   a = acceleration

velocity (v) $= \frac{dx}{dt}$

Acceleration (a) $= \frac{dv}{dt} = \frac{d^2x}{dt^2} = \frac{dv}{dx}.v$

Newton's second law of motion = $F = ma = m\frac{dv}{dt} = mv\frac{dv}{dx}$

3] Simple Electric Circuits :-

a) $i = \frac{dq}{dt}$ or $q = \int i\,dt$.

b) Voltage drop across resistance = Ri

c) " " " inductance = $L\frac{di}{dt}$

d) " " " capacitance = $\frac{q}{C}$

e) Kirchoff's law = $L\frac{di}{dt} + Ri = E$

A] Circuit involving

a) L and R with voltage source and E in series $\boxed{i = \frac{E}{R} + ce^{-Rt/L}}$

b) R and C " " " " " " $i = \left[\frac{E}{R} - \frac{q_0}{RC}\right]e^{-t/RC}$

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4] Heat flow:-

$$q = -KA\frac{dT}{dx}$$

where $A = 2\pi x$. (area).

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Chapter 3 – Fourier Series:

1] $$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$a_0 = \frac{1}{\pi}\int_{c}^{c+2\pi} f(x)\,dx \qquad a_n = \frac{1}{\pi}\int_{c}^{c+2\pi} f(x)\cos nx\,dx$$

$$b_n = \frac{1}{\pi}\int_{c}^{c+2\pi} f(x)\cdot\sin nx\,dx$$

2] **for $c=0$**

$$a_0 = \frac{1}{\pi}\int_{0}^{2\pi} f(x)\,dx \qquad a_n = \frac{1}{\pi}\int_{0}^{2\pi} f(x)\cdot\cos nx\,dx.$$

$$b_n = \frac{1}{\pi}\int_{0}^{2\pi} f(x)\cdot\sin nx\,dx.$$

3] **For even and odd functions:** **Range: $-\pi$ to π**

a) For ~~odd~~ even function
$f(-x) = +f(x)$

$$a_0 = \frac{2}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$a_n = \frac{2}{\pi}\int_{-\pi}^{\pi} f(x)\cos nnx\,dx \qquad b_n = 0$$

b) for ~~even~~ odd function $f(-x) = f(x)$ $f(-x) = -f(x)$

$a_0 = 0 \quad a_n = 0$

$$b_n = \frac{2}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx.$$

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Note:- $\int uv\,dx = uv_1 - u'v_2 + u''v_3 - u'''v_4 + \ldots$

$\sin n\pi = 0 \quad \sin 2n\pi = 0$

$\cos n\pi = (-1)^n \quad \cos 2n\pi = 1$

4] For interval between 0 to 2L

$$a_0 = \frac{1}{L}\int_0^{2L} f(x)\,dx \qquad a_n = \frac{1}{L}\int_0^{2L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$b_n = \frac{1}{L}\int_0^{2L} f(x)\sin\frac{n\pi x}{L}\,dx$$

5] For interval between $-L \leq x \leq L$

1] for even ~~odd~~ function

$f(-x) = +f(x)$

$$a_0 = \frac{2}{L}\int_0^{L} f(x)\,dx \qquad a_n = \frac{2}{L}\int_0^{L} f(x)\cos\frac{n\pi x}{L}$$

2] for odd ~~even~~ function

$f(-x) = -f(x)$

$$b_n = \frac{2}{L}\int_0^{L} f(x)\cdot\sin\frac{n\pi x}{L}$$

6] Half Range Cosine Expansion

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos\frac{n\pi x}{L}$$

$$a_0 = \frac{2}{L}\int_0^{L} f(x)\,dx$$

$$a_n = \frac{2}{L}\int_0^{L} f(x)\cos\frac{n\pi x}{L}\,dx.$$

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C] Half Range Sine Expansion

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$$

$$b_n = \frac{2}{L} \int_0^L \sin \frac{n\pi x}{L} f(x)\,dx.$$

D] Harmonic Analysis:

$$a_0 = 2 \times \frac{\Sigma y}{m}$$

$$a_n = \frac{2 \times \Sigma y \cos nx}{m}$$

$$b_n = \frac{2 \times \Sigma y \sin nx}{m}$$

$\therefore$ m = total number or members in 'y'

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Chapter 4 – Reduction formulae, Beta & Gamma Functions.

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1] For $\int \sin^n x\,dx$

$$I_n = \frac{n-1}{n} I_{n-2}$$

2] For n = positive integer

$$\int_0^{\pi/2} \sin^n x\,dx = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdots\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2} \qquad n=\text{even}$$

$$= \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdots\frac{4}{5}\cdot\frac{2}{3}\cdot 1 \qquad n=\text{odd}$$

$$\int_0^{\pi/2} \sin^n x\,dx = \int_0^{\pi/2} \cos^n x\,dx.$$

Additional Results:

$$\int_0^{\pi} \sin^n x\,dx = 2\int_0^{\pi/2} \sin^n x\,dx$$

$$\int_0^{\pi} \cos^n x\,dx = 2\int_0^{\pi/2} \cos^n x\,dx \qquad n=\text{even}$$

$$= 0 \qquad n=\text{odd.}$$

$$\int_0^{2\pi} \sin^n x\,dx = 4\int_0^{\pi/2} \sin^n x\,dx \qquad n=\text{even}$$

$$= 0 \qquad n=\text{odd}$$

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3] $\int_0^{\pi/2} \sin^m x \cos^n x \, dx$

$$= \frac{\{(m-1)(m-3)\dots 2 \text{ or } 1\}\{(n-1)(n-3)\dots 2 \text{ or } 1\}}{(m+n)(m+n-2)(m+n-4)\dots 2 \text{ or } 1} \times P$$

$P = \pi/2$ m and n are even

$= 1$ for all values of m and n

B] Gamma Functions.

$$\Gamma n = \int_0^\infty e^{-x} x^{n-1} dx$$

Properties.

1] $\Gamma n = 2\int_0^\infty e^{-x^2} x^{2n-1} dx$

2] $\Gamma 1 = 1$

3] $\Gamma(n+1) = n\Gamma n$ if n = fraction

4] $\Gamma 0 = \infty$

5] $\Gamma(n+1) = n!$ if n = integer

6] $\Gamma\frac{1}{2} = \sqrt{\pi}$

7] $\int_0^\infty e^{-ky} y^{n-1} dy = \frac{\Gamma n}{k^n}$

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c] Beta Functions

$$B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$$

Properties.

1) $B(m,n) = B(n,m)$

2] $B(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cdot \cos^{2n-1}\theta \, d\theta$

3) $B\left[\frac{p+1}{2}, \frac{q+1}{2}\right] = 2\int_0^{\pi/2} \sin^p\theta \cos^q\theta \, d\theta$

4] $B(m,n) = \frac{\Gamma m \, \Gamma n}{\Gamma m+n}$

5] $\Gamma p \, \Gamma 1+p = \frac{\pi}{\sin p\pi}$

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Chapter-5- Differential Under Integral Sign

Rule I -

$$I(\alpha) = \int_a^b f(x,\alpha)\,dx \quad \text{then} \quad \frac{dI}{d\alpha} = \int_a^b \frac{\partial f(x,\alpha)}{\partial \alpha}\,dx$$

Rule-II -

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha}\int_{a(\alpha)}^{b(\alpha)} f(x,\alpha)\,dx = \int_{a(\alpha)}^{b(\alpha)} \frac{\partial}{\partial \alpha} f(x,\alpha)\,dx + f(b,\alpha)\frac{db}{d\alpha} - f(a,\alpha)\frac{da}{d\alpha}$$

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Chapter 5 – Differential Under Integral Sign and Error Function.

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Error function:

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du$$

1] Complementary Error Function:

$$erfc(x) = \frac{2}{\sqrt{\pi}} \int_0^\infty e^{-u^2} du.$$

2] Alternative defination:

$$erf(x) = \frac{1}{\sqrt{\pi}} \int_0^{x^2} e^{-t}\, t^{-1/2} dt.$$

Properties of Error Function:

1) $erf(\infty) = 1$
2) $erf(0) = 0$
3) $erf(x) + erfc(x) = 1$
4) $erf(-x) = -erf(x)$
5) $erf(x) = \frac{2}{\sqrt{\pi}} \left[x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + \cdots \right]$

Diff of Error Function:

$$\frac{d}{dx} erf(ax) = \frac{2ae^{-a^2x^2}}{\sqrt{\pi}}$$

Integration of Error function

$$\int_0^t erf(ax)dx = t\, erf(at) + \frac{1}{a\sqrt{\pi}} e^{-a^2t^2} - \frac{1}{a\sqrt{\pi}}$$

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Chapter-6-Curve Tracing and Rectification of Curves

I] Tracing of cartesian curves

a) Symmetry about X-axis : powers of y are even everywhere

b) Symmetry about Y axis : powers of x are even everywhere

c) Symmetry about both axis : powers of x and y are even "

d) Symmetry about opposite quadrants : Equation remains unchanged if x and y are changed to $-x$ and $-y$

e) Symmetry about y-x line : Equation remains unchanged if x is replaced with y and y is replaced with x

2] Point of Intersection

a) with X-axis : Put $y=0$ and obtain values of x
$(a,0)$ $(a_1,0)$ are point of intersection

b) with Y-axis : Put $x=0$ and obtain values of y
$(0,a_1)$ $(0,a_2)$ are point of intersection

c) with origin : Put $x=0$ and $y=0$

3] Tangent at origin

If curve passes through origin, then find tangent at origin with equating lowest degree term

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4] Special point

$\frac{dy}{dx} = \pm \infty$ — parallel to Y axis

$\frac{dy}{dx} > 0$ curve increasing

$\frac{dy}{dx} < 0$ curve decreasing

5] Asymptote

a) parallel to X axis: Equate co-efficient of highest power of x to zero, we get asymptote parallel to x-axis

b) parallel to Y axis: Equate co-efficient of highest power of y to zero, we get asymptote parallel to y-axis

6] Region of Absence

a) for $y^2 = f(x)$: Suppose $y^2 < 0$ and find values of x

b) for $x^2 = f(y)$: Suppose $x^2 < 0$ and find values of y.

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II] Tracing of Polar Form

A] Symmetry

i) Initial line $\theta=0$: Replace θ by $-\theta$, if equation remains unchanged then curve is symmetric

ii) About Pole: Replace r by $-r$, if given equation remains unchanged then curve is symmetric about pole

iii] About $\theta=\pi/2$: Replace θ by $(\pi-\theta)$, if equation remains unchanged curve is symmetric about $\theta=\pi/2$

iv] If equation remains unchanged by changing θ to $-\theta$ and r to $-r$, then curve is symmetric about line $\theta=\pi/2$ through pole perpendicular to initial line.

B] Rose Curves.

$r=a\sin n\theta$ or $r=a\cos n\theta$

1] For $r=a\sin n\theta$, first loop is drawn along $\theta=\frac{\pi}{2n}$

2] For $r=a\cos n\theta$, first loop is drawn along $\theta=0$

3] If n is odd there are 'n' number of loops

4) If n is even there are '2n' number of loops

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Chapter-7 - Co-ordinate System, Plane, Straight line.

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I] Relations Between Three Co-ordinate Systems:

a) Relation between cartesian and spherical polar system of coordinates

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

b] Relation between cartesian and cylindrical system of coordinates

$$x = \rho\cos\theta$$
$$y = \rho\sin\phi$$
$$z = z$$

c] For spherical polar coordinates (r, θ, ϕ)

$$r = \sqrt{x^2+y^2+z^2}$$
$$\cos\theta = \frac{z}{r}$$
$$\tan\phi = \frac{y}{x}$$

d] For cylindrical coordinates (ρ, ϕ, z)

$$\rho = \sqrt{x^2+y^2}$$
$$\tan\phi = \frac{y}{x}$$
$$z = z$$

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II] Distance Formula:

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

III] Division of Join of Given Points

Internally in ratio $m:n$

$$x = \frac{mx_2+nx_1}{m+n} \qquad y = \frac{my_2+my_1}{m+n} \qquad z = \frac{mz_2+nz_1}{m+n}$$

Externally in ration $m:n$

$$x = \frac{mx_2-mx_1}{m-n} \qquad y = \frac{my_2-ny_1}{m-n} \qquad z = \frac{mz_2-mz_1}{m-n}$$

IV] Direction Cosines (Relation)

$$l^2+m^2+n^2=1$$

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$$

V] Direction Ratios:

If a,b,c are drs in $\frac{l}{a}=\frac{m}{b}=\frac{n}{c}$ then dc ares

$$l = \frac{a}{\sqrt{a^2+b^2+c^2}} \qquad m = \frac{b}{\sqrt{a^2+b^2+c^2}} \qquad n = \frac{c}{\sqrt{a^2+b^2+c^2}}$$

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vi] Angle

$(l_1 m_1 n_1)$ $(l_2 m_2 n_2)$ = direction cosines

$$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$$

$$\cos\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

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SPHERE

I] Center Radius Form:-

center $= (a, b, c)$

radius $= r$

$\therefore$ Equation: $(x-a)^2+(y-b)^2+(z-c)^2=r^2$

Note: If center lies at origin equation becomes:

$x^2+y^2+z^2=r^2$ which is the standard form.

II] General form:

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0$$

where center $= (-u, -v, -w)$

radius $= \sqrt{u^2+v^2+w^2-d}$

III] Intercept Form:

If sphere cuts x axis at $x=a$

y axis at $y=b$

z axis at $z=c$

$u=-\frac{a}{2}$ $\quad v=-\frac{b}{2}$ $\quad w=-\frac{c}{2}$

- Intercept form Eq^n: $x^2+y^2+z^2-ax-by-cz=0$

IV] Diameter Form:

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)+(z-z_1)(z-z_2)=0$$

Touching Spheres:

If spheres touch

a) externally then $d=r_1+r_2$

b) Internally then $d=r_1-r_2$ $\quad$ (d = distance between

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INTERSECTION OF PLANE AND SPHERE

$$S = x^2+y^2+z^2+2ux+2vy+2wz+d=0$$

$$u = lx+my+nz+p=0$$

Equation: $S+\lambda U=0$

$$\text{Length of perpendicular} = \left| \frac{lu+mv+nw+p}{\sqrt{l^2+m^2+n^2}} \right|$$

where u, v, w are centers of sphere
l, m, n are points of contact on plane

ORTHOGONAL SPHERES

$$2u_1u_2+2v_1v_2+2w_1w_2=d_1+d_2$$

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CONE:

Fixed point = vertex (α, β, γ)

Given curve = guiding curve or base

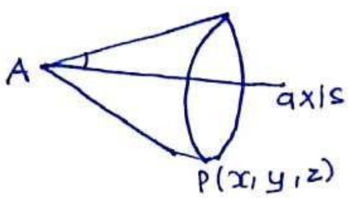
Any straight line = generator

Direction Ratios (l, m, n)

$\therefore$ Equation of generator passing through (α, β, γ) with direction ratios l, m, n is given by

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$$

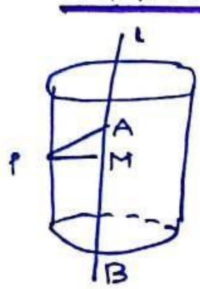
RIGHT CIRCULAR CONE:



Angle between generator AP and axis is given as

$$\cos\alpha = \frac{l(x-\alpha)+m(y-\beta)+n(z-\gamma)}{\sqrt{l^2+m^2+n^2}\sqrt{(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2}}$$

RIGHT CIRCULAR CYLINDER



$P = (x, y, z)$ = any point on cylinder

$A = (\alpha, \beta, \gamma)$ = fixed point on axis AB

PM = Radius of cylinder $= r$

$AP = \sqrt{(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2}$

AM = Projection of AP on axis AB

$$= \frac{l(x-\alpha)+m(y-\beta)+n(z-\gamma)}{\sqrt{l^2+m^2+n^2}}$$

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Chapter No 10 - Multiple Integrals.

I] Double Integration:

a] when x_1 and x_2 are functions of y and y_1 and y_2 are constant, then we integrate first wrt x keeping y constant and then integrate y between y_1 and y_2

$$\int_{y_1=a}^{y_2=\alpha} \left[\int_{x_1=f(y)}^{x_2=f(y)} f(x,y)\,dx \right] dy$$

b] when y_1 and y_2 are functions of x and x_1 and x_2 are constants, then we integrate first wrt y keeping x constant and then integrate x between x_1 and x_2

$$\int_{x_1=a}^{x_2=\alpha} \left[\int_{y_1=f(x)}^{y_2=f(x)} f(y,x)\,dy \right] dx.$$

c] x_1 x_2 and y_1 y_2 are constant limits then region of integration is rectangle. So we can use the given order of integration

d] If $f(x,y) = h(x) \cdot g(y)$ and if both limits are constant the

$$\int_a^b \int_c^d f(x,y)\,dx\,dy = \int_a^b h(x)\,dx \int_c^d g(y)\,dy.$$

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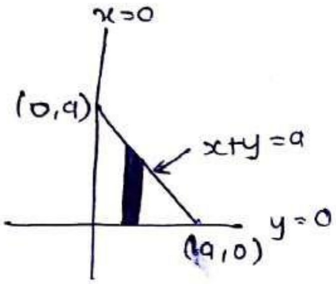
II] Evaluation of Double Integrals Without Limits:

a] Integrating first w.r.t y then x.

- Draw region r where $x=0$ $y=0$ and $x+y=a$.
- For this integration consider vertical strip



- For $x=0$ $y=a$ and for $y=0$ $x=a$

∴ Points of intersection are $(0,a)$ & $(a,0)$

- Limits for integration of vertical strip is given as

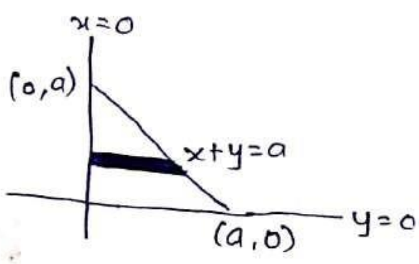
$x=0$ to a

$y=0$ to $a-x$

- $I = \int_0^a \int_0^{a-x}$

b] Integrating first wrt x then y

- Draw region r where $x=0$ $y=0$ and $x+y=a$.
- For this integration consider horizontal strip.



- For $x=0$ $y=a$ and for $y=0$ $x=a$

∴ Points of intersection are $(0,a)$ $(a,0)$

- Limits for integration of horizontal Strip is

$x=0$ to $a-y$

$y=0$ to a

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c] Double Integration in Polar coordinate form

a] Function is always integrated first wrt r and then wrt θ.

ii] The strip is always radial and it is taken from pole.

iii] Rotate strip in anticlockwise direction.

iv] $x = r\cos\theta \quad y = r\sin\theta$

$$x^2 + y^2 = r^2$$

$$dx\,dy = r\,dr\,d\theta$$

v] For First Quadrant $= \theta = 0$ to $\pi/2$

First, Second " $= \theta = 0$ to π

First, Second, Third $= \theta = 0$ to $3\pi/2$

1st, 2nd, 3rd, 4th $= \theta = 0$ to 2π.

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TRIPLE INTEGRATION:

- If function $f(x,y,z)$ then integrate 1st wrt 'z' and then wrt 'y' and lastly with respect to x.
- Limits of z are in terms of x and y
- Limits of y are in terms of x
- Limits of x are always constants.

$$\int_a^b \int_{f_1(x)}^{f_2(x)} \int_{\phi_1(x,y)}^{\phi_2(x,y)} f(x,y,z)\,dz\,dy\,dx =$$

$$\int_a^b \left\{ \int_{f_1(x)}^{f_2(x)} \left[\int_{\phi_1(x,y)}^{\phi_2(x,y)} f(x,y,z)\,dz \right] dy \right\} dx.$$

TRIPLE INTEGRATION IN SPHERICAL POLAR COORDINATES.

$$x^2+y^2+z^2=r^2$$

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

$$dx\,dy\,dz = r^2\sin\theta\,dr\,d\theta\,d\phi$$

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AREA

1. $\int y\,dx \rightarrow$ Area bounded on x axis
2. $\int x\,dy \rightarrow$ Area bounded on y axis
3. $\iint dx\,dy \rightarrow$ Area is in cartesian form
4. $\iint r\,dr\,d\theta \rightarrow$ Area is in polar form.

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